INSTITUTO SUPERIOR DE TRANSPORTES E COMUNICAÇÕES

MiniTeste 3 de
ÁLGEBRA LINEAR E GEOMETRIA ANALÍTICA
Curso: Licenciatura em Engª Electrotécnica — **Data:** 05/04/2024
Turma: LEE11 — **Pontuação:** 100Pts
Docentes: Amade Monteiro — **Duração:** 50 minutos

Questões

1. **[40 Pontos]** Pelo desenvolvimento da primeira linha, calcule o determinante de $A = \begin{pmatrix} -1 & 2 & 3 & -4 \\ 4 & 2 & 0 & 0 \\ -1 & 2 & -3 & 0 \\ 2 & 5 & 3 & 1 \end{pmatrix}$,

2. **[30 Pontos]** Pelo método de Gauss-Jordan, determine a inversa da Matriz $A = \begin{bmatrix} 3 & 0 & 2 \\ 9 & 1 & 7 \\ 1 & 0 & 1 \end{bmatrix}$

3. **[30 Pontos]** Aplicando o método da adjunta, determine a inversa da Matriz $A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

Bom Trabalho!

Gabarito MT3 - ALGA I 2024

1. $A = \begin{pmatrix} -1 & 2 & 3 & -4 \\ 4 & 2 & 0 & 0 \\ -1 & 2 & -3 & 0 \\ 2 & 5 & 3 & 1 \end{pmatrix}$

$$|A| = -1\cdot(-1)^{1+1}\cdot \det(A_{11}) + 2\cdot(-1)^{1+2}\cdot \det(A_{12}) + 3\cdot(-1)^{1+3}\det(A_{13}) +$$
$$+ (-4)\cdot(-1)^{1+4}\cdot \det(A_{14})$$

$$\det(A_{11}) = \begin{vmatrix} 2 & 0 & 0 \\ 2 & -3 & 0 \\ 5 & 3 & 1 \end{vmatrix} = 2\cdot\begin{vmatrix} -3 & 0 \\ 3 & 1 \end{vmatrix} - \cancel{0\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix}}^{\,0} + \cancel{0\cdot\begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix}}^{\,0}$$
$$= 2\cdot(-3) - 0 + 0 = -6$$

$$\det(A_{12}) = \begin{vmatrix} 4 & 0 & 0 \\ -1 & -3 & 0 \\ 2 & 3 & 1 \end{vmatrix} = 4\cdot\begin{vmatrix} -3 & 0 \\ 3 & 1 \end{vmatrix} - \cancel{0\cdot\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}}^{\,0} + \cancel{0\cdot\begin{vmatrix} -1 & -3 \\ 2 & 3 \end{vmatrix}}^{\,0} = -12$$

$$\det(A_{13}) = \begin{vmatrix} 4 & 2 & 0 \\ -1 & 2 & 0 \\ 2 & 5 & 1 \end{vmatrix} = 4\cdot\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix} - 2\cdot\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} + \cancel{0\cdot\begin{vmatrix} -1 & 2 \\ 2 & 5 \end{vmatrix}}^{\,0} =$$
$$= 4\cdot 2 - 2\cdot(-1) + 0 = 10$$

$$\det(A_{14}) = \begin{vmatrix} 4 & 2 & 0 \\ -1 & 2 & -3 \\ 2 & 5 & 3 \end{vmatrix} = 4\begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix} - 2\cdot\begin{vmatrix} -1 & -3 \\ 2 & 3 \end{vmatrix} + \cancel{0\cdot\begin{vmatrix} -1 & 2 \\ 2 & 5 \end{vmatrix}}^{\,0}$$
$$= 4\cdot 21 - 2\cdot 3 + 0 = 78$$

logo $\det(A) = -1\cdot\det(A_{11}) + (-2)\cdot\det(A_{12}) + 3\cdot\det(A_{13}) + 4\cdot\det(A_{14})$

$\det(A) = (-1)\cdot(-6) - 2\cdot(-12) + 3\cdot 10 + 4\cdot 78 = 372$

2. $A = \begin{bmatrix} 3 & 0 & 2 \\ 9 & 1 & 7 \\ 1 & 0 & 1 \end{bmatrix}$

$$[A|I] = \left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 9 & 1 & 7 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] L_1 = l_{1/3} \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & 1/3 & 0 & 0 \\ 9 & 1 & 7 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right].$$

$$\begin{matrix} L_2 = l_2 - 9l_1 \\ L_3 = l_3 - l_1 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 1 & 1 & -3 & 1 & 0 \\ 0 & 0 & 1/3 & -1/3 & 0 & 1 \end{array}\right] L_3 = l_3 \,(x)\, 3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 1 & 1 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 3 \end{array}\right] \begin{matrix} L_1 = l_1 - \frac{2}{3} l_3 \\ L_2 = l_2 - l_3 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & -1 & 0 & 3 \end{array}\right]$$

Logo $A^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & -3 \\ -1 & 0 & 3 \end{bmatrix}$ #

3. $A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det(A)} \cdot Adj(A)$$

$$\det(A) = \left| \begin{array}{ccc:cc} 2 & 1 & 3 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2 \\ 5 & 1 & 3 & 5 & 1 \end{array} \right| = 12 + 5 + 0 - (30 + 2 + 0)$$

$$\det(A) = 17 - 32 = -15$$

$$Adj(A) = C^t$$

$$C = \begin{bmatrix} \hat{C}_{11} & \hat{C}_{12} & \hat{C}_{13} \\ \hat{C}_{21} & \hat{C}_{22} & \hat{C}_{23} \\ \hat{C}_{31} & \hat{C}_{31} & \hat{C}_{33} \end{bmatrix}$$

$$\hat{C}_{11} = (-1)^2 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5$$

$$\hat{C}_{12} = (-1)^3 \cdot \begin{vmatrix} 0 & 1 \\ 5 & 3 \end{vmatrix} = +5$$

$$\hat{C}_{13} = (-1)^4 \cdot \begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} = -10$$

$$\hat{C}_{21} = (-1)^3 \cdot \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = -(3-3) = 0$$

$$\hat{C}_{22} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 5 & 3 \end{vmatrix} = 6 - 15 = -9$$

$$\hat{C}_{23} = (-1)^5 \cdot \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} = -(2-5) = 3$$

Continua

$$\hat{C}_{31} = (-1)^4 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1-6 = -5$$

$$\hat{C}_{32} = (-1)^5 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} = -2$$

$$\hat{C}_{33} = (-1)^6 \cdot \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 4$$

$$C = \begin{bmatrix} 5 & 5 & -10 \\ 0 & -9 & 3 \\ -5 & -2 & 4 \end{bmatrix}$$

$$\mathrm{Adj}(A) = C^t = \begin{bmatrix} 5 & 0 & -5 \\ 5 & -9 & -2 \\ -10 & 3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \mathrm{Adj}(A)$$

$$A^{-1} = -\frac{1}{15} \cdot \begin{bmatrix} 5 & 0 & -5 \\ 5 & -9 & -2 \\ -10 & 3 & 4 \end{bmatrix} = \begin{bmatrix} -5/15 & 0 & 5/15 \\ -5/15 & 9/15 & 2/15 \\ 10/15 & -3/15 & -4/15 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/3 & 0 & 1/3 \\ -1/3 & 3/5 & 2/15 \\ 2/3 & -1/5 & -4/15 \end{bmatrix}$$