INSTITUTO SUPERIOR DE TRANSPORTES E COMUNICAÇÕES

MiniTeste 1 de

ÁLGEBRA LINEAR E GEOMETRIA ANALÍTICA

Curso: Licenciatura em Engª Electrotécnica — **Data:** 08/03/2024

Turma: LEE11 — **Pontuação:** 100Pts

Docente: Amade Monteiro — **Duração:** 50 minutos

Questões

1. **[10 Pontos]** Resolva a equação seguinte em C:
   $x^2 - 2x + 10 = 0$

2. **[15 Pontos]** determine $m \in R$ de modo que $z = (m^2 - 5m + 6) + (m^2 - 4)i$ seja Imaginário puro

3. **[5 + 10 Pontos]** Se $z = 4 + 4i$ e $w = 3 - 5i$, calcule:
   a) $z - w$ b) $\frac{z}{w}$

4. **[10 Pontos]** Obtenha o complexo z de modo que $2z + zi - (1 - i) - 4 = 3i$

5. **[15 + 10 Pontos]** Considere $z = -5 - 5i$
   a) Escreva o número na forma trigonométrica;
   b) Calcule z^4

6. **[10 + 15 Pontos]** Dados os complexos $z = \sqrt{2}\,(\cos 45° + i\operatorname{sen} 45°)$ e $w = 2(\cos 15° + i\operatorname{sen} 15°)$, calcule:

   a) $\frac{z}{w}$ b) $\sqrt[3]{z}$

Bom Trabalho!

Correção mT_1 _ ALGA _ 2024

1.

$$x^2-2x+10=0$$

$$\Delta=b^2-4ac=(-2)^2-4.10.1=-36$$

$$x_{1/2}=\frac{-b\pm\sqrt{\Delta}}{2.a}$$

$$x_{1/2}=\frac{2\pm\sqrt{36}}{2.1}$$

$$x_{1/2}=\frac{2\pm 6i}{2} \to x_1=\frac{2-6i}{2}=1-3i, \quad x_2=\frac{2+6i}{2}=1+3i$$

sol: $x=\{1-3i,\ 1+3i\}$

2.

$$Z=(m^2-5m+6)+(m^2-4)i$$

Para que Z seja imaginário puro

$$\begin{cases} m^2-5m+6=0 \\ m^2-4\neq 0 \end{cases} \Rightarrow \begin{cases} (m-2)(m-3)=0 \\ (m-2)(m+2)\neq 0 \end{cases}$$

$$\Rightarrow \begin{cases} m=2 \vee m=3 \\ m\neq 2 \wedge m\neq -2 \end{cases}$$

sol: $m=3$.

3. $Z = 4+4i$ e $W = 3-5i$

a) $Z - W = 4+4i-(3-5i) = 4-3+4i+5i = 1+9i$

b) $\frac{Z}{W} = \frac{(4+4i)(3+5i)}{(3-5i)(3+5i)}$

$= \frac{12+20i+12i-20}{9+25} = \frac{-8+32i}{34}$

$= -\frac{8}{34} + \frac{32}{34}i = -\frac{4}{17} + \frac{16}{17}i$

4.

$2z + zi - (1-i) - 4 = 3i$

$z(2+i) - 1 + i - 4 = 3i$

$z(2+i) = 3i - i + 5$

$z(2+i) = 2i + 5$

$z = \frac{2i+5}{2+i}$

$z = \frac{(2i+5)(2-i)}{(2+i)(2-i)} = \frac{4i+2+10-5i}{4+1}$

$z = \frac{12-i}{5} = \frac{12}{5} - \frac{i}{5}$

5. $z = -5 - 5i$

a) $z = r(\cos\theta + i \operatorname{sen}\theta)$

$a = -5 \quad b = -5$

$r = \sqrt{a^2 + b^2} = \sqrt{(-5)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$

$\theta = \pi + \arctan\left(\left|\frac{b}{a}\right|\right)$

$\theta = \pi + \arctan\left(\left|\frac{-5}{-5}\right|\right) = \pi + \operatorname{arctg} 1$

$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$z = r(\cos\theta + i\operatorname{sen}\theta) = 5\sqrt{2}\left[\cos\frac{5\pi}{4} + i \operatorname{sen}\frac{5\pi}{4}\right]$

b) $z^4 = ?$

$z^n = r^n . (\cos n\theta + i \operatorname{sen} n\theta)$

$z^4 = (5\sqrt{2})^4 . \left(\cos 4 . \frac{5\pi}{4} + i \operatorname{sen} 4 . \frac{5\pi}{4}\right)$

$z^4 = (5\sqrt{2})^4 . (\cos 5\pi + i \operatorname{sen} 5\pi)$

6. $Z = \sqrt{2}\,(\cos 45° + i \operatorname{sen} 45°)$

$W = 2\,(\cos 15° + i \operatorname{sen} 15°)$

a) $\frac{Z}{W} = \frac{\sqrt{2}}{2} \cdot \left[\cos(45°-15°) + i \operatorname{sen}(45°-15°)\right]$

$= \frac{\sqrt{2}}{2}\,(\cos 30° + i \operatorname{sen} 30°)$

b) $\sqrt[3]{z} = ?$

$\sqrt[n]{z} = r^{\frac{1}{n}} \cdot \left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i \operatorname{sen}\left(\frac{\theta + 2k\pi}{n}\right)\right], k = 0 \ldots, n-1$

$\sqrt[3]{z} = \left(\frac{\sqrt{2}}{2}\right)^{\frac{1}{3}} \cdot \left[\cos\left(\frac{\frac{\pi}{4} + 2k\pi}{3}\right) + i \operatorname{sen}\left(\frac{\frac{\pi}{4} + 2k\pi}{3}\right)\right]$

$k = 0, 1, 2$

Para $k = 0$

$Z_0 = \sqrt[3]{\frac{\sqrt{2}}{2}} \cdot \left(\cos \frac{\pi}{12} + i \operatorname{sen} \frac{\pi}{12}\right)$

$k = 1 \Rightarrow Z_1 = \sqrt[3]{\frac{\sqrt{2}}{2}} \cdot \left[\cos\left(\frac{\frac{\pi}{4} + 2\pi}{3}\right) + i \operatorname{sen}\left(\frac{\frac{\pi}{4} + 2\pi}{3}\right)\right]$

$Z_1 = \sqrt[3]{\frac{\sqrt{2}}{2}} \cdot \left[\cos\left(\frac{3\pi}{4}\right) + i \operatorname{sen}\left(\frac{3\pi}{4}\right)\right]$

$k = 2 \Rightarrow Z_2 = \sqrt[3]{\frac{\sqrt{2}}{2}} \cdot \left[\cos\left(\frac{\frac{\pi}{4} + 4\pi}{3}\right) + i \operatorname{sen}\left(\frac{\frac{\pi}{4} + 4\pi}{3}\right)\right]$

$Z_2 = \sqrt[3]{\frac{\sqrt{2}}{2}} \cdot \left[\cos\left(\frac{17}{12}\pi\right) + i \operatorname{sen}\left(\frac{17}{12}\pi\right)\right]$

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